Assume the given graph is directed and has no negative cycle.

Create a new vertex S with one edge to each existing vertex. (edge weight can be arbitrary)

Use Bellman-Ford algorithm to find shortest paths from S to all vertices. (Shortest paths exist because the graph has no negative cycle)

Let d[u] be the length of a shortest path from S to u, then for each edge (u, v), d[u]+w(u, v)>=d[v].

The new weight is w’(u, v)=w(u, v)+d[u]-d[v]>=0.

Since the new weight is non-negative, Dijkstra's algorithm can be used to find shortest paths.